



Condensation characteristics inside a vertical tube considering the presence of mass transfer, vapor velocity and interfacial shear

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Received 4 September 2000; received in revised form 22 February 2001

Abstract

Condensation heat transfer in a vertical tube has been investigated analytically by considering the vapor flow and mass transfer. Condensation processes are divided into two sections: the forming film section and the formed film section. The present analysis takes into account the effects of momentum transfer that results from mass transfer and vapor flow on the interfacial shear and hence condensation. Two major parameters that govern the condensation heat transfer the relative velocity ratio B and the momentum transfer factor U , along with a comprehensive factor ω that includes mass transfer, the vapor velocity and the thermophysical properties, are proposed to describe the condensation mechanism. It is shown that the effects of interfacial shear and mass transfer are strongly dependent on the thermophysical properties of the working fluids and the operating temperatures. The theoretical predictions are in good agreement with the experimental data from other investigations for the upward vapor flow in water cases. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Condensation; Mass transfer; Interfacial shear; Vapor velocity; Comprehensive factor

1. Introduction

The first film condensation model was proposed by Nusselt [1]. It assumes negligible effects of the interfacial shear and the vapor flow. Improvements and modifications to Nusselt's theoretical solution have been made by a number of researchers [2–10]. Rohsenow, Webber, Lehtinen and others considered the effect of interfacial shear but assumed constant interfacial shear along the axial direction (cf. [2]). Minkowycz and Sparrow [3] included the role of subcooling, interface, and diffusion based on Nusselt's problem of a vertical plate. Chen [4] accounted for the effect of surface waves and turbulence in turbulent film condensation. Whereas Seban, Hodgson and Faghri [5,6] deemed that the effect of interfacial

shear was very small for condensation in a vertical tube, the studies of reflux condensation in a two-phase closed thermosyphon [7–10] showed that the effects of the interfacial shear and mass transfer on film condensation are too strong to be neglected.

The two contrary opinions in regard to the significance of the interfacial shear and mass transfer on the film condensation reflect that the system under consideration is complex and has yet to be understood. In addition, the previous solutions to Nusselt's problem are not applicable to the vertical tube situation because the vapor velocity in the tube diminishes with its flow.

The purpose of this paper is to further explore the influence of interfacial shear on condensation in a vertical tube. In what follows, a mathematical model that accounts for the interfacial shear and mass transfer is developed based on the first principles. Nonlinear dimensionless condensation equations are derived, and solved by a finite difference method. The model prediction is compared with Nusselt's solution. Then, two

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Nomenclature		x	axial coordinate
B	relative velocity ratio	<i>Greek symbols</i>	
F	Fanning number	Γ	mass flow rate per unit width
Gr	two-phase Grashof number	δ	film thickness
Ja	Jakob number	μ	dynamic viscosity
m''	mass flux of phase change	ρ	density
Nu	Nusselt number	φ	defined in Eq. (13b)
R	radius	τ	shear stress
S	dimensionless axial coordinate	Δ	temperature difference
U	momentum transfer factor	ν	kinematic viscosity
v	y -component velocity	ω	comprehensive factor
y	transverse coordinate	<i>Subscripts</i>	
c_p	specific heat	c	net film flux
g	gravitational acceleration	i	interface
h_{fg}	latent heat of vaporization	l	liquid
l	length of condenser section	R	ratio
N_R	Nusselt number ratio	v	vapor
Pr	Prandtl number	io	interface without mass transfer
Re	Reynolds number	L	condenser length
T	temperature	N	Nusselt's solution
u	x -component velocity	w	wall

controlling parameters, the relative velocity ratio B and the momentum transfer factor U , are proposed to study the mechanism underlying the condensation process. A comprehensive factor ω that lumps the effects of mass transfer, vapor velocity and thermophysical properties is employed to explore the significance of interfacial shear on the condensation process. Finally, the solutions predicted by this work are compared with the experimental data from other investigations. The present work considers both downward and upward vapor flows, which affect interfacial shear differently, and the variation of vapor and mass transfer along the vapor flow direction.

2. Condensation model with the interfacial shear

2.1. The motion of liquid film

For the motion of liquid film as shown in Fig. 1, the following assumptions will be made throughout this analysis: (1) Since the film thickness is less than the tube radius for this work, the curvature effect is negligible and the governing equations of boundary layer type for a vertical plate can be applied. (2) The liquid film flow is laminar. (3) The fluid inertia term in the momentum equation can be neglected. (4) The temperature distribution inside the film is linear. (5) All thermophysical properties of vapor and film are constant. (6) Condensation is a steady state process.

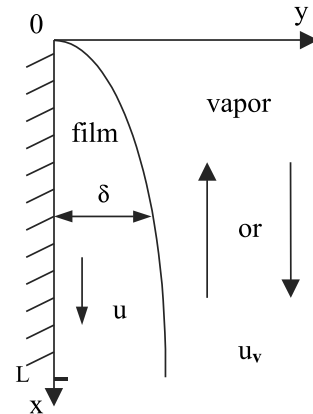


Fig. 1. A model of condensation.

Based on the above assumptions, conservation of mass, momentum and energy in the film yields the following governing equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\mu_l \frac{\partial^2 u}{\partial y^2} + (\rho_l - \rho_v)g = 0, \quad (2)$$

$$\frac{\partial^2 T}{\partial y^2} = 0. \quad (3)$$

The associated boundary conditions are

$$y = 0 :$$

$$u = v = 0, \tag{4a}$$

$$T = T_v. \tag{4b}$$

$$y = \delta :$$

$$\mu_1 \frac{\partial u}{\partial y} = \tau_i, \tag{5a}$$

$$k_1 \frac{\partial T}{\partial y} = m'' h_{fg} = \frac{\partial \Gamma}{\partial x} h_{fg}, \tag{5b}$$

$$u = u_i, \tag{5c}$$

$$T = T_v. \tag{5d}$$

$$x = 0 :$$

$$u = 0, \tag{6a}$$

$$\delta = 0 \text{ and for the upward vapor flow,} \tag{6b}$$

$$u_v = 0. \tag{6c}$$

$$x = L : \text{for the downward vapor flow, } u_v = 0. \tag{7}$$

Eq. (2) is integrated, and Eqs. (4b) and (5a) are applied

$$u = \frac{g}{\nu_1} \left(y\delta - \frac{y^2}{2} \right) + \frac{\tau_i y}{\mu_1}, \tag{8}$$

$$\Gamma = \int_0^\delta \rho_1 u dy = \frac{g\rho_1 \delta^3}{3\nu_1} + \frac{\tau_i \delta^2}{2\nu_1}. \tag{9}$$

Eq. (9) can also be written as

$$\tau_i = \frac{2\nu_1 \Gamma}{\delta^2} - \frac{2\rho_1 g \delta}{3}. \tag{10}$$

Substitution of Eq. (10) in (8) there results

$$u = \frac{2\Gamma y}{\rho_1 \delta^2} + \frac{gy\delta}{3\nu_1} - \frac{gy^2}{2\nu_1}, \tag{11a}$$

and

$$u_i = u|_{y=\delta} = \frac{2\Gamma}{\rho_1 \delta} - \frac{\delta^2 g}{6\nu_1}, \tag{11b}$$

and τ_w can be obtained as

$$\tau_w = \mu \frac{\partial u}{\partial y} \Big|_{y=0} = \frac{3\Gamma \nu_1}{\delta^2} - \frac{\tau_i}{2}. \tag{12}$$

2.2. Mass transfer and vapor flow

To account for the influence of mass transfer due to the momentum exchange of phase change, the following modification factor [11], a ratio of the interfacial shear with mass transfer to the interfacial shear without mass transfer, is used

$$U = \tau_i / \tau_{i0} = \varphi / [\exp(\varphi) - 1], \tag{13a}$$

where

$$\varphi = - \frac{2m''}{f \rho_v |u_v - u_i|}, \tag{13b}$$

$$\tau_{i0} = \frac{1}{2} f \rho_v (u_v - u_i)^2, \tag{13c}$$

$$\tau_i = \frac{1}{2} f \rho_v (u_v - u_i)^2 U. \tag{14}$$

According to the assumption (4a) and (4b) and the boundary condition (5b)

$$m'' = \frac{k_1 \Delta T}{h_{fg} \delta}.$$

Hence

$$\varphi = - \frac{2k_1 \Delta T}{f \rho_v h_{fg} \delta |u_v - u_i|}, \tag{15}$$

where f is the pure Fanning friction factor,

$$f = C Re_v^n \tag{16}$$

with

$$Re_v = \frac{2\rho_v R}{\mu_v} |u_v - u_i|. \tag{17}$$

C and n are given in Table 1.

Since there are two types of vapor flows in a tube, namely the upward vapor flow and the downward vapor flow, the vapor flow should be discussed separately, as shown in Fig. 2.

Table 1
Values of C and n

Re_v range	C	n
$Re_v < 2000$	16	-1
$2000 < Re_v < 4000$	1/1525	0.33
$4000 < Re_v < 3 \times 10^4$	0.079	-0.25
$3 \times 10^4 < Re_v < 1 \times 10^6$	0.046	-0.2

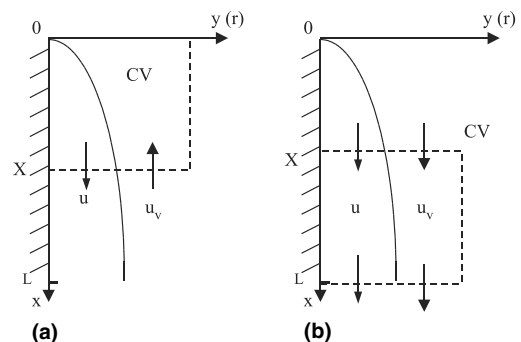


Fig. 2. Control volume: (a) upward vapor, (b) downward vapor.

(1) The upward vapor flow: At $x = 0$, the film begins to form, and the vapor is fully condensed hypothetically

$$u_v|_{x=0} = 0, \Gamma|_{x=0} = 0.$$

For the control volume (CV), a mass conservation equation is

$$\pi(R - \delta)^2 \rho_v u_v = 2\pi R \Gamma.$$

The vapor flow velocity is written as

$$u_v = \frac{2\Gamma}{\rho_v R}. \tag{18}$$

(2) The downward vapor flow: At $x = L$, it is assumed that the vapor is fully condensed, that is, $u_v|_{x=L} = 0$, $\Gamma|_{x=L} = \Gamma_L$.

From the mass conservation for CV, one has

$$\begin{aligned} \pi(R - \delta)^2 \rho_v u_v + 2\pi R \Gamma &= 2\pi R \Gamma_L, \\ u_v &= \frac{2(\Gamma_L - \Gamma)}{\rho_v R}. \end{aligned} \tag{19}$$

Set Γ_c as a net effluent liquid mass flux from the control volume,

$$\Gamma_c = \begin{cases} \Gamma & \text{upward vapor flow,} \\ \Gamma_L - \Gamma & \text{downward vapor flow,} \end{cases} \tag{20}$$

Hence

$$u_v = \frac{2\Gamma_c}{\rho_v R}. \tag{21}$$

2.3. Dimensionless equations

The following variables are introduced to dimensionalize the governing equations:

$$\begin{aligned} u^* &= \frac{u}{\delta_N^2 g / \nu_1}, \quad S = \frac{x}{L}, \quad \delta^* = \frac{\delta}{R}, \quad \Gamma^* = \frac{\Gamma}{\mu_1}, \\ \tau^* &= \frac{\tau}{\rho_1 g \delta_N}, \quad y^* = \frac{y}{\delta}, \end{aligned}$$

$$Gr = \left(\frac{8gR^3}{\nu_1^2} \right)^{1/2}, \quad Ja = \frac{c_{pl} \Delta T}{h_{fg}}, \quad Pr_1 = \frac{c_{pl} \mu_1}{k_1},$$

$$\rho^* = \frac{\rho_v}{\rho_l}, \quad \mu^* = \frac{\mu_v}{\mu_l}, \quad R^* = \frac{R}{L},$$

where

$$\Delta T = T_v - T_w, \quad \delta_N = \left(\frac{4Jav_1^2 x}{Pr_1 g} \right)^{1/4}.$$

The dimensionless form of Eqs. (11a), (11b) and (21) are respectively given by

$$u^* = \frac{16\Gamma^* y^*}{Gr^2 \delta^{*2} \delta_N^2} + \frac{y^* \delta^{*2}}{3\delta_N^2} - \frac{\delta^{*2} y^{*2}}{2\delta_N^2}, \tag{22a}$$

$$u_i^* = \frac{16\Gamma^*}{Gr^2 \delta_N^2 \delta^*} - \frac{\delta^{*2}}{6\delta_N^2}, \tag{22b}$$

$$u_v^* = \frac{16\Gamma_c^*}{Gr^2 \delta_N^2 \rho^*}. \tag{23}$$

Thus

$$u_v^* - u_i^* = \frac{16\Gamma_c^*}{Gr^2 \delta_N^2 \rho^*} B, \tag{24}$$

where

$$B = 1 - \frac{u_i^*}{u_v^*}. \tag{25}$$

The dimensionless form of Eqs. (16), (15), (14), (12) and (9) are expressed as, respectively

$$f = C \left(\frac{4\Gamma^* B}{\mu^*} \right)^n, \tag{26}$$

$$\varphi = - \frac{Ja}{f \Gamma_c^* B Pr_1 \delta^*}, \tag{27}$$

$$\tau_i^* = \frac{16\omega}{Gr^2 \delta^{*2} \delta_N^2}, \tag{28}$$

$$\tau_w^* = \frac{24\Gamma^*}{Gr^2 \delta^{*2} \delta_N^2} + \frac{\tau_i^*}{2}, \tag{29}$$

$$\Gamma^* = \frac{\delta^{*3} Gr^2}{24} + \omega, \tag{30}$$

where ω is defined as a comprehensive factor affected by the interfacial shear stress.

$$\omega = \pm \frac{\Gamma_c^{*2} B^2 f \delta^{*2}}{\rho^*} U \quad \begin{cases} \text{downward shear,} \\ \text{upward shear.} \end{cases} \tag{31}$$

In the light of the assumption (5a)–(5d) the dimensionless form of the boundary condition (5b) can be written as

$$\frac{\partial \Gamma^*}{\partial S} \frac{R^* Pr_1}{Ja} = \frac{1}{\delta^*}, \quad \text{or } \delta^* = \left(\frac{Ja}{R^* Pr_1} \right) / \left(\frac{\partial \Gamma^*}{\partial S} \right). \tag{32}$$

There exist no analytical solutions and linear relations between S and other variables for above equations. However the solutions can be obtained numerically by a finite-difference method since Γ^* is a nonlinear single-valued function of S .

3. Results and discussions

Fig. 3 shows the heat transfer ratio N_R , defined as the ratio of Nusselt number obtained in the present study to that predicted by Nusselt, namely

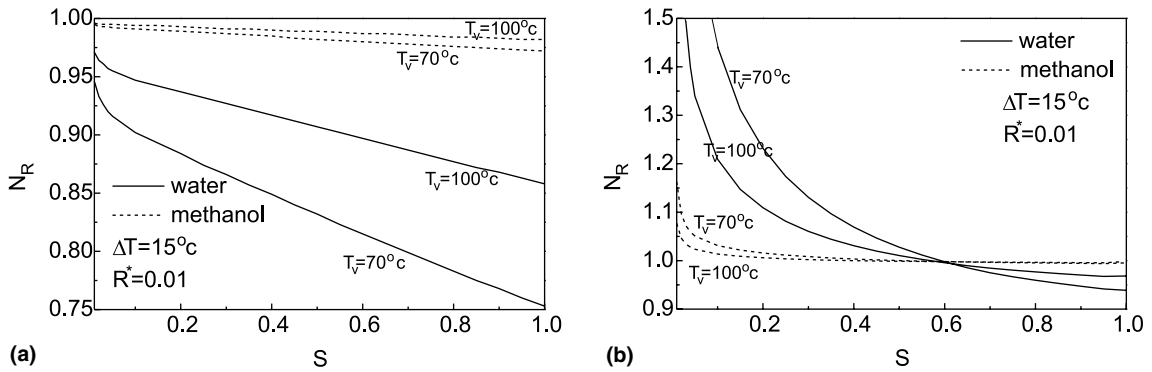


Fig. 3. N_R under different fluids and temperatures: (a) upward vapor flow, (b) downward vapor flow.

$$N_R = Nu/Nu_N = \frac{\delta_N^*}{\delta^*}$$

The second equality is derived since $Nu = x/\delta = S/\delta^* R^*$.

In other words, N_R also expresses the ratio of film thickness predicted by Nusselt’s solution to that of the present study. All the cases reported in the present work are calculated for $L = 1$ m.

According to the rate of change of condensation variables, the condensation process along the direction of film flow, is divided into two sections, a forming film section ($0 \leq S \leq 0.2$) and a formed film section ($S > 0.2$). In the forming film section, where the film begins to form, the effect of the interfacial shear on condensation heat transfer is more significant. After forming the film ($S > 0.2$), the film layer is gradually thickened along its flow direction, and the rate of change of both the vapor velocity and mass transfer tend to be moderate. This is the formed film section, where the effect of the interfacial shear is strongly dependent on the working liquid properties, the operation temperatures and some other parameters.

Since the interfacial shear is mainly affected by mass transfer and the vapor velocity, it is expected that these two parameters pay an important role in condensation heat transfer. Mass transfer can be described by em-

ploying the momentum transfer factor U , as defined in Eqs. (13a)–(13c). As shown in Fig. 4, the factor U has significant change under two cases. One is taken place in the forming film section where phase-change heat transfer occurs much intensively due to the condensation incipience so that the effect of mass transfer becomes strong. Another case occurs at the low relative vapor Reynolds number Re_v ($Re_v < 2000$ or Re_v is in the range of laminar), in which the velocity difference between the vapor and the film is so much small that the interfacial shear without mass transfer τ_{io} becomes very small. In this time, as long as the interfacial shear with mass transfer τ_i increases a little, as defined in Eqs. (13a)–(13c), U will be augmented largely. Here, it should be pointed out that the factor U only reflects the relative effect of mass transfer on condensation instead of the strength of mass transfer. In the forming film section, for example, the strength of mass transfer is certainly stronger for downward film flow than for the upward film flow, but the relative effect of mass transfer or the value of U is less for the downward film flow than the upward film flow. It is also seen that the influence of the properties of working fluid on mass transfer is considerably small.

Another major factor, by which the influence of the vapor velocity on the interface and condensation is

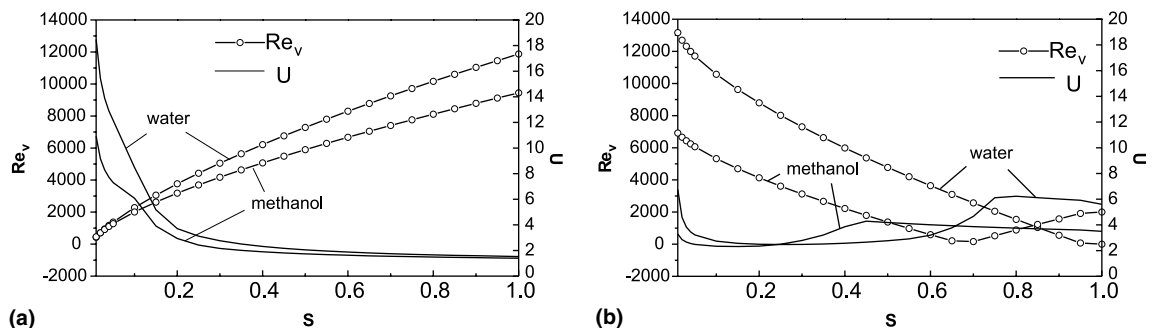


Fig. 4. Relation between Re_v and U : (a) upward vapor flow, (b) downward vapor flow.

reflected, is the relative velocity ratio B , as defined in Eq. (25). It is physically defined a ratio of the relative vapor velocity to the absolute vapor velocity. Fig. 5 exhibits the variation of vapor velocity. If the ratio B approaches 1, it is implied that the effect of the vapor velocity on the interfacial shear is relatively strong. In fact, the largest interfacial shear occurs at the entrance of vapor where the vapor velocity is the highest. The value of B is always larger than 1 for the upward vapor flow, meaning that a negative interfacial shear results in obstructing the film flow. For the downward vapor flow, B is less than 1. When $0 < B < 1$, the interfacial shear becomes positive, which is available for the film flow and condensation. But when $B < 0$, the interfacial shear turns to be negative which is no longer available for condensation. The variation of vapor velocity is strongly subject to the thermophysical properties and the operating temperatures. It is seen that the effect of vapor velocity is much stronger in the water cases than in the methanol cases. This is because the film viscosity and the density ratio of liquid to vapor are larger for water. Hence, the interfacial shear or the vapor drag in the water cases modifies the liquid flow comparatively more.

The comprehensive factor ω , as defined in Eq. (31), including the effect of mass transfer, the effect of vapor velocity and the properties of the liquid, collectively expresses the influence of interfacial shear on condensation. When the mass transfer and interfacial shear are absent, ω is equal to zero, and Eq. (30) becomes Nusselt's solution. On the other hand, ω represents an additional film flux affected by the interfacial shear. When the absolute value of ω is small, it implies that the effect of interfacial shear is not significant, or the discrepancy between the prediction of the present work and that of Nusselt's solution becomes very small. Thus, the factor ω has two explanations, which not only describes the behavior of the interfacial shear but also reflects the variation trend of the film flux. For example, the negative value of ω is expressive of the decrease of film flux due to the negative additional film flux, and also denotes that the interfacial shear is negative so that the film flow is modified by the vapor drag, as shown in Fig. 6. It is noted that the absolute value of ω is much smaller in the methanol cases than in the water cases. Comparing Figs. 3 with 6, it is found that when the absolute value of ω is less than 10, the relative error $(1 - N_R)$ is less than $\pm 5\%$,

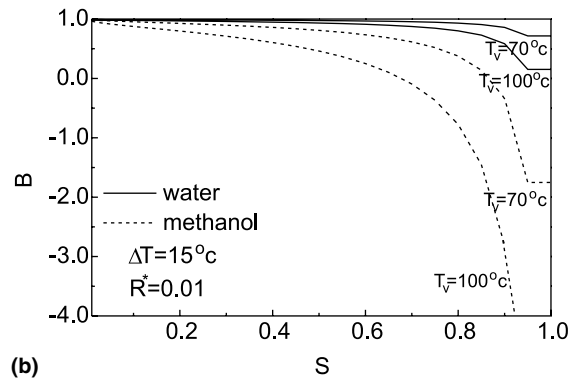
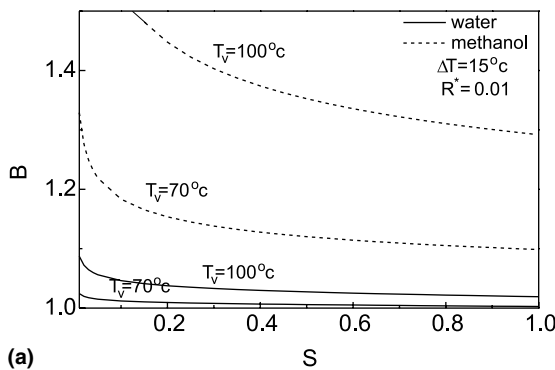


Fig. 5. B under different fluids and temperatures: (a) upward vapor flow, (b) downward vapor flow.

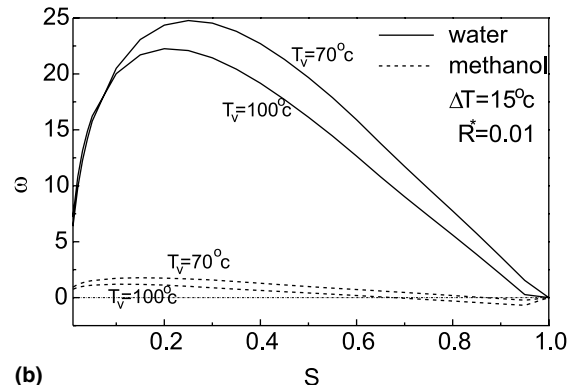
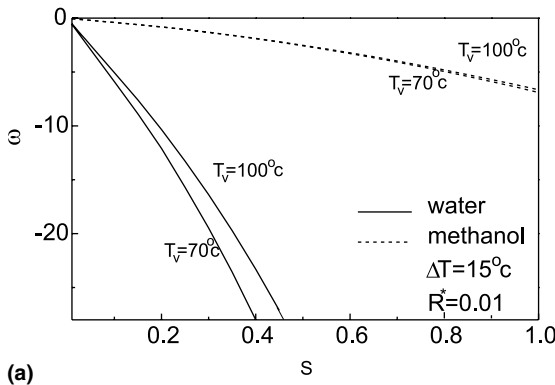


Fig. 6. ω under the different fluids and temperatures: (a) upward vapor flow, (b) downward vapor flow.

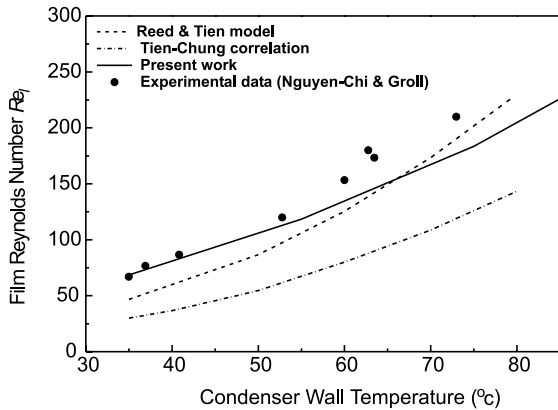


Fig. 7. Experimental data of Nguyen-Chi and Groll [12] and theoretical prediction from the present work (working fluid is water, $R = 0.0085$ m, $l_c = 1.00$ m).

which means the effect of interfacial shear is very small so that it can be neglected. If the relative error $\pm 5\%$ is taken as a criterion to estimate whether the effect of interfacial shear is significant or not, the effect of interfacial shear on condensation can be neglected for the methanol or other similar liquids, and Nusselt's solution can be applicable for these working liquids cases. However, in the water cases, the effect of interfacial shear should be considered due to the present of strong vapor drag, particularly, for the upward vapor flow.

Fig. 7 illustrates heat pipe, in which the vapor flows upward, flooding data for water reported on the basis of the condenser wall temperature [12]. The data presented are from a heat pipe operating at an inclination angle of 10° from vertical, the closet case to vertical operation. It can be seen that all of the theoretical curves [8,13] underpredict the experimental data. For the present work, the curve is closer to the experimental data at the low wall temperature than at the high temperature. The agreement between the experimental data and the theoretical predictions of this analysis is very good if the effect of inclination angle is considered.

4. Conclusions

By the theoretical analysis, a mathematical model on condensation heat transfer, including the effect of the interfacial shear, mass transfer and the vapor velocity, has been carried out. Nonlinear dimensionless equations have been derived, and solved numerically.

The results indicated that there exists a discrepancy between the present model prediction and Nusselt's solution. The discrepancy is strongly subject to the thermophysical properties of the working fluids, the operating temperatures and other parameters.

According to the characteristics of condensation, the condensation procedure is divided into two sections, the forming film section and the formed film section.

The effect of interfacial shear is strongly dependent on the mass transfer and vapor velocity. Mass transfer described by the momentum transfer factor U has the significant effect in the forming film section and in the low vapor Reynolds number Re_v . The relative velocity ratio B influences the interfacial shear strongly when it approaches 1. The comprehensive factor ω , including the effects of mass transfer, the vapor velocity and the thermophysical properties, can collectively describe the effect of interfacial shear. If the relative error $(1 - N_R)$ is less than $\pm 5\%$, the effect of interfacial shear can be neglected. Otherwise, it should be considered.

The theoretical solutions predicted by the present work agree very well with the experimental data for the upward vapor flow.

Acknowledgements

The author is grateful for the support of this work by the Natural Science Foundation of Jiangxi Province in China under contract 986029. Discussions and comments from Prof. A. Nakayama of Shizuoka University in Japan were very helpful to the conduct of this work.

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